10-3 day 1 Polar Functions

Learning Objectives:

I can convert between Cartesian and polar coordinates.

I can graph relations defined by a polar equation

I can convert polar equation into parametric equations and graph the curve

I can find the slope of a polar curve

I can write the equation of a tangent line to a polar curve

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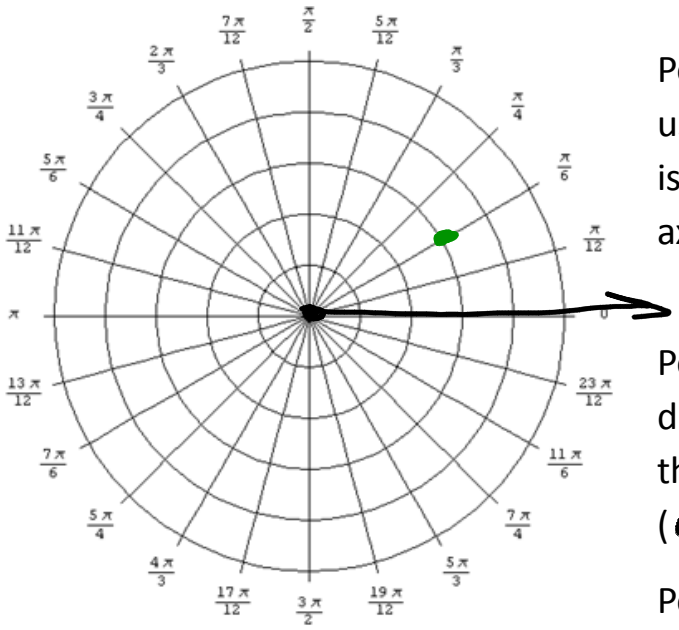

Points can be located in the plane using polar coordinates. The origin is called the Pole. The positive x – axis is called the initial ray.

Points are described by their distance away from the pole (r) and the angle away from the initial ray (θ).

Points – $(r;\theta)$

θ

$(r;\theta)$

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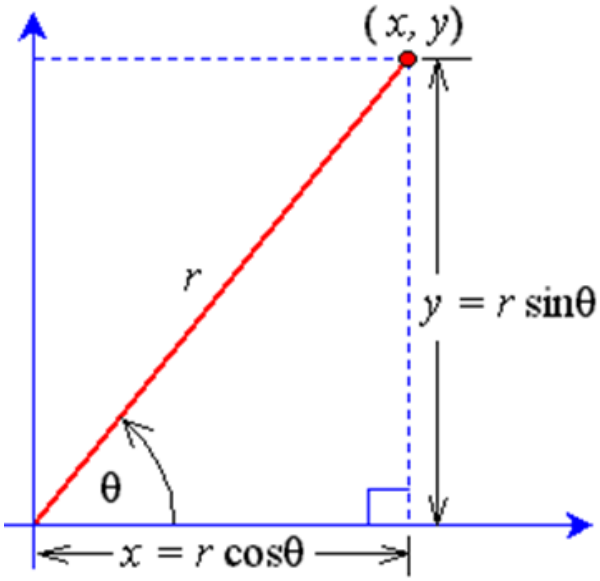

$$x = r \cdot \cos(\theta)$$

$$y = r \cdot \sin(\theta)$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

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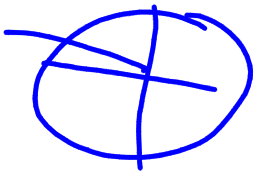
Ex1. Convert $(-3\sqrt{3}, 3)$ to polar coordinates

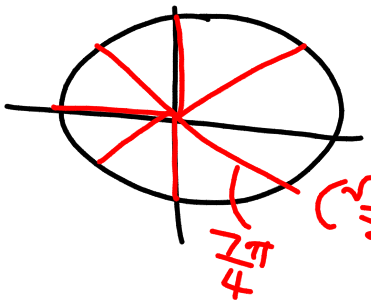
$r = \sqrt{(-3\sqrt{3})^2 + (3)^2} = \sqrt{36} = 6$

$\theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{5\pi}{6}$



$\left(6, \frac{5\pi}{6}\right)$

Ex2. Convert $\left(4;\frac{7\pi}{4}\right)$ to rectangular coordinates



$x = r\cos\theta \quad y = r\sin\theta$

$x = 4\cos\frac{7\pi}{4} \quad y = 4\sin\frac{7\pi}{4}$

$x = 2\sqrt{2} \quad y = -2\sqrt{2}$

$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ $(2\sqrt{2}, -2\sqrt{2})$

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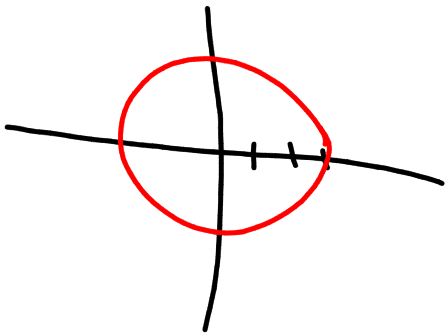
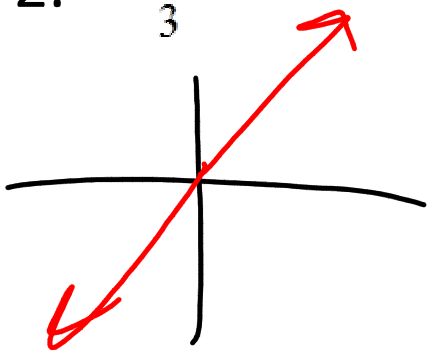
Ex3. Graph each polar equation w/o a G.C.

1. r=3



2. $\theta = \frac{\pi}{3}$



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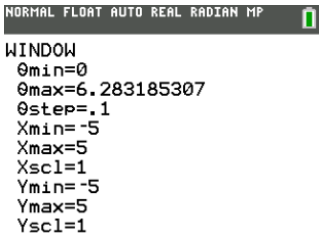
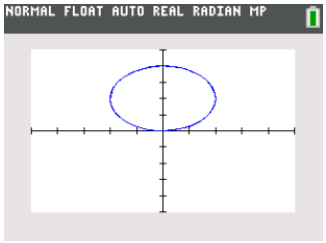
Ex4. Graph each polar equation with a G.C.

circles

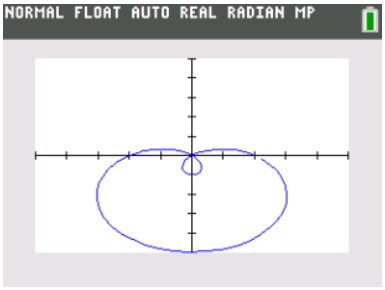
1. $r = 4\sin(\theta)$





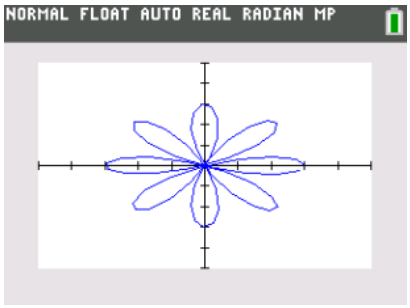
Limaçon

2. $r = 2 - 3\sin(\theta)$

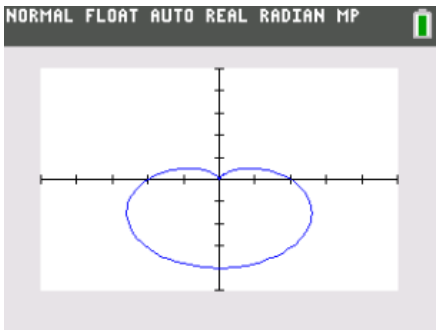


rose

3. $r = 3\cos(4\theta)$



4. $r = 2 - 2\sin(\theta)$



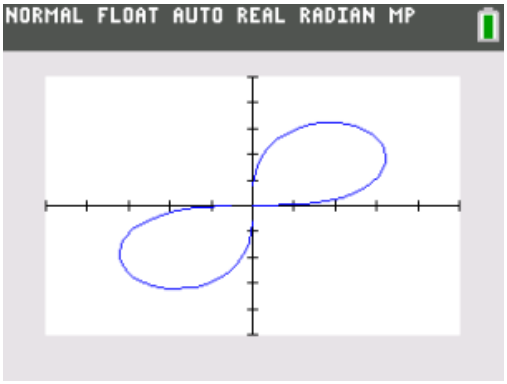
cardiod

5. $r^2 = 16\sin(2\theta)$

$r = \pm\sqrt{16\sin(2\theta)}$

$r = \sqrt{16\sin(2\theta)}$

Lemniscate



Graphing Polar Equations Parametrically

The polar equation $r = f(\theta)$ can be graphed parametrically as:

$$x = r \cdot \cos(t) \qquad x = f(t) \cdot \cos(t)$$

$$y = r \cdot \sin(t) \qquad y = f(t) \cdot \sin(t)$$

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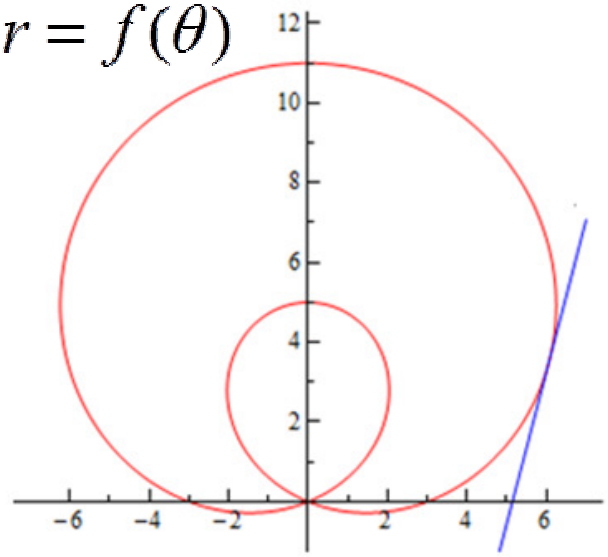
Slopes of Polar Curves

$r = f(\theta)$



$$x = f(t)\cdot\cos(t)$$

$$y = f(t)\cdot\sin(t)$$

$$\frac{dx}{dt} = f'(t)\cos(t) - f(t)\sin(t)$$

$$\frac{dy}{dt} = f'(t)\sin(t) + f(t)\cos(t)$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

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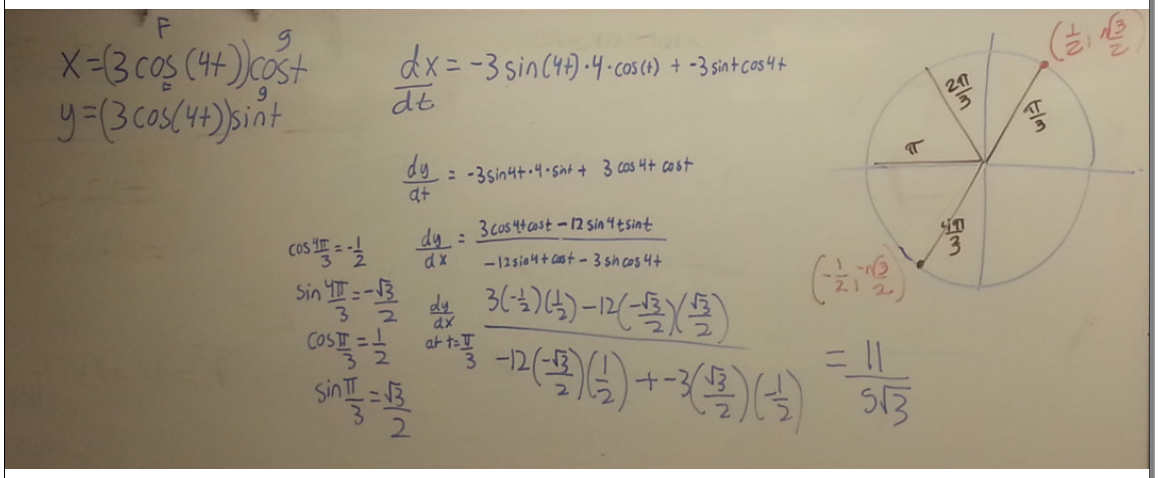
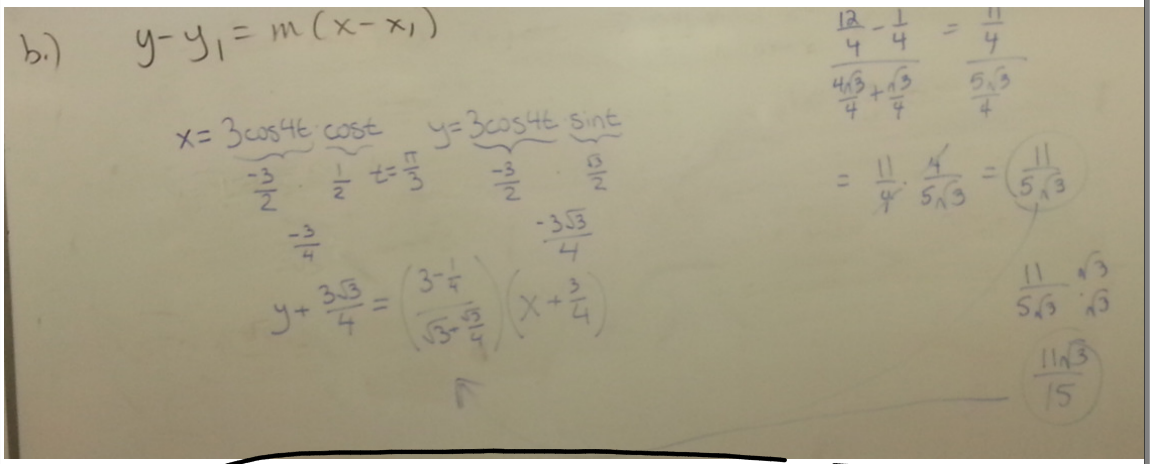
Ex5. Use the polar equation $r = 3\cos(4\theta)$ to answer the following questions.

a.) Find the slope of the tangent line at $\theta = \frac{\pi}{3}$



b.) Write the equation of the tangent line to the curve at $\theta = \frac{\pi}{3}$.



$$y + \frac{3\sqrt{3}}{4} = \frac{11\sqrt{3}}{15}\left(x + \frac{3}{4}\right)$$

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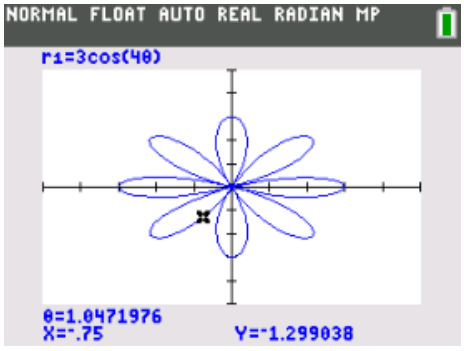
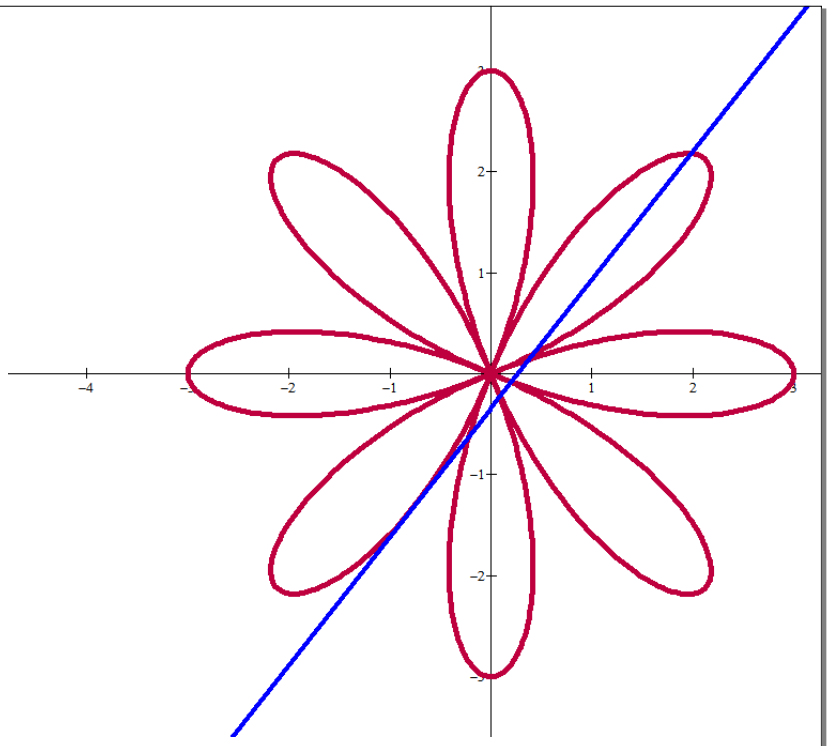

NORMAL FLOAT AUTO REAL RADIAN MP
r1=3cos(4θ)
θ=1.0471976
X=-.75
Y=-1.299038


-4
-2
-1
1
2
2
1
-1
-2

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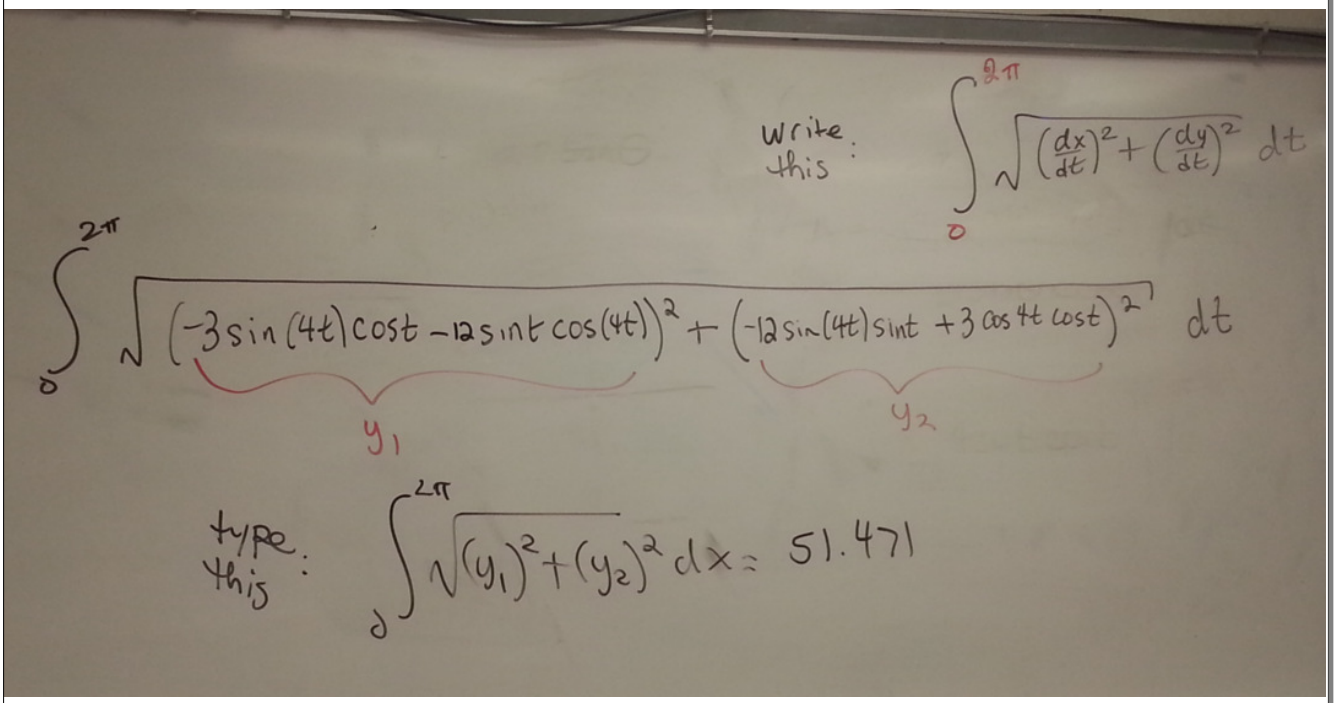
c.) Find the length of the polar curve



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Homework

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36, 39-42, 61, 65, 66

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